

DCS-Final Project

**Digital Controller Design
Analysis
with RCM Mechanism
System**

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Summary

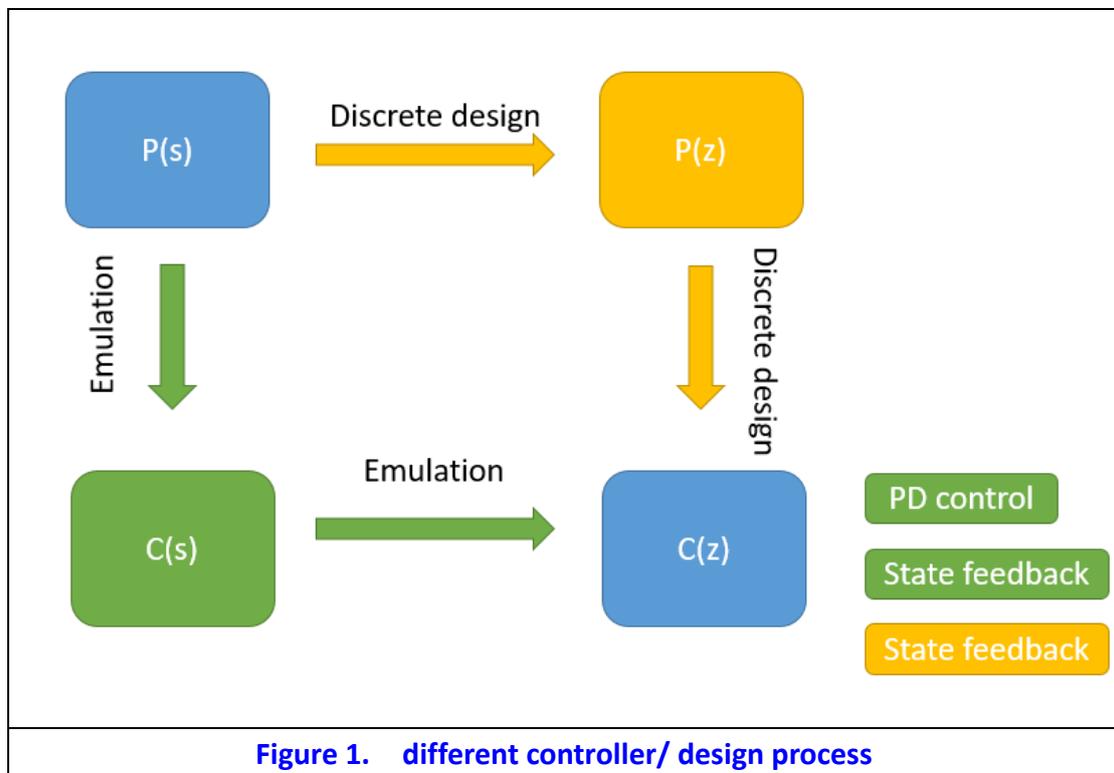


Figure 1. different controller/ design process

In this Final Project, we want to design our digital controller- PD controller and state feedback poleplacement method by discrete and emulation method(Figure 1).

According to the simulation result, we analysis the performance by the comparison of the different :

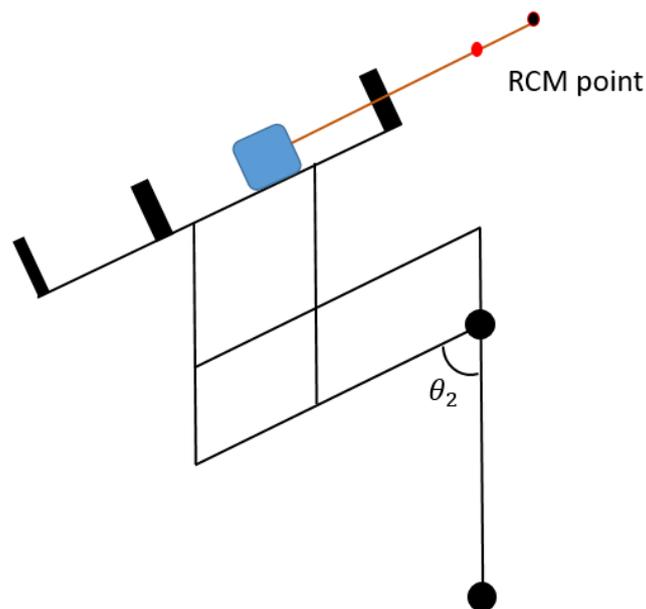
Controller

Design process

Sampling time

System

We want to analysis a robotic system with a remote center of motion (RCM). The RCM mechanism have been applied to many medical industry widely. Microsurgery is a famous example, and a well-known surgery robot-da Vinci Surgical System is also applied this technique. The RCM point will be static when the robot moves, which is RCM mechanism's feature. We demand this mechanism to have high precision and high speed to avoid any injury to patient.



As the result, we require the degree of θ_2 must have

Specification

0 steady state error

settling time(t_s) in 0.1s

Overshoot (M_p) in 1%

According to Newton's second law of motion rotation form: $\tau = I\ddot{\theta}$, $I = mr^2$

$$\tau_1 = I \cdot \ddot{\theta}_1$$

$$\tau_2 - \frac{l_1}{2} \sin \theta_2 \cdot 2 \cdot m_1 - l_1 \sin \theta_2 \cdot m_2 - l_3 \sin \theta_2 \cdot m_2 - (x + x_0) \sin \theta_2 \cdot M - l_{ee} \sin \theta_2 \cdot m_{ee} = I \cdot \ddot{\theta}_2$$

$$I = \sum_{i=0}^n m_i r_i^2 = \left(\frac{l_1}{2} \sin \theta_2\right)^2 \cdot m_1 \cdot 2 + (l_1 \sin \theta_2)^2 \cdot m_2 + (l_3 \sin \theta_2)^2 \cdot m_2 + [(x + x_0) \sin \theta_2]^2 \cdot M + (l_{ee} \sin \theta_2)^2 \cdot m_{ee}$$

$$I = \sin^2(\theta_2) \left(\frac{l_1^2}{2} m_1 + l_1^2 m_2 + l_3^2 m_2 + [(x + x_0)]^2 \cdot M + l_{ee}^2 \cdot m_{ee} \right)$$

$$\tau_2 - \sin \theta_2 (l_1 \cdot m_1 + l_1 \cdot m_2 + l_3 \cdot m_2 + (x + x_0) \cdot M + l_{ee} \cdot m_{ee}) = I \cdot \ddot{\theta}_2$$

$$\text{Let } (l_1 \cdot m_1 + l_1 \cdot m_2 + l_3 \cdot m_2 + (x + x_0) \cdot M + l_{ee} \cdot m_{ee}) = A,$$

$$\left(\frac{l_1^2}{2} m_1 + l_1^2 m_2 + l_3^2 m_2 + [(x + x_0)]^2 \cdot M + l_{ee}^2 \cdot m_{ee} \right) = B$$

$$\tau_1(t) = B \cdot \sin^2(\theta_2) \cdot \ddot{\theta}_1$$

$$\tau_2(t) = B \cdot \sin^2(\theta_2) \cdot \ddot{\theta}_2 + A \cdot \sin \theta_2$$

Let

$$x_1(t) = \theta_1(t)$$

$$x_2(t) = x_1(t) = \theta_1(t)$$

$$x_3(t) = \theta_2(t)$$

$$x_4(t) = x_3(t) = \theta_2(t)$$

$$\tau_1(t) = B \cdot \sin^2(x_3(t)) \cdot x_2(t)$$

$$\tau_2(t) = B \cdot \sin^2(x_3(t)) \cdot x_4(t) + A \cdot \sin x_3(t)$$

$$x_1(t) = x_2(t)$$

$$x_2(t) = \frac{\tau_1(t)}{B \cdot \sin^2(x_3(t))}$$

$$x_3(t) = x_4(t)$$

$$x_4(t) = \frac{\tau_2(t) - A \cdot \sin x_3(t)}{B \cdot \sin^2(x_3(t))}$$

This is a nonlinear system, so we linearize the system in the operating point

$$\tau_1(t) = 0$$

$$\tau_2(t) = A \sin\left(\frac{\pi}{2}\right) = A$$

$$x_1(t) = 0$$

$$x_2(t) = 0$$

$$x_3(t) = \frac{\pi}{2}$$

$$x_4(t) = 0$$

$$\Delta x_1(t) = \Delta x_2(t)$$

$$\begin{aligned} \Delta x_2(t) &= \frac{\partial \frac{\tau_1(t)}{B \cdot \sin^2(x_3(t))}}{\partial \tau_1(t)} \Big|_{\substack{\tau_1(t)=0 \\ x_3(t)=\frac{\pi}{2}}} \cdot \Delta \tau_1(t) + \frac{\partial \frac{\tau_1(t)}{B \cdot \sin^2(x_3(t))}}{\partial x_3(t)} \Big|_{\substack{\tau_1(t)=0 \\ x_3(t)=\frac{\pi}{2}}} \cdot \Delta x_3(t) \\ &= \frac{1}{B} \Delta \tau_1(t) \end{aligned}$$

$$\Delta x_3(t) = \Delta x_4(t)$$

$$\begin{aligned} \Delta x_4(t) &= \frac{\partial \frac{\tau_2(t) - A \cdot \sin x_3(t)}{B \cdot \sin^2(x_3(t))}}{\partial \tau_2(t)} \Big|_{\substack{\tau_2(t)=A \\ x_3(t)=\frac{\pi}{2}}} \cdot \Delta \tau_2(t) + \frac{\partial \frac{\tau_2(t) - A \cdot \sin x_3(t)}{B \cdot \sin^2(x_3(t))}}{\partial x_3(t)} \Big|_{\substack{\tau_2(t)=A \\ x_3(t)=\frac{\pi}{2}}} \\ &\quad \cdot \Delta x_3(t) = \frac{1}{B} \Delta \tau_2(t) \end{aligned}$$

Continuous-time state space model

Apply

$$l_0 = 97.6mm$$

$$l_1 = 112.08mm$$

$$l_2 = 167.5mm$$

$$l_3 = 69mm$$

$$l_{ee} = 128.61mm$$

$$s = 104mm$$

$$x_0 = 22mm$$

$$0 < x < s$$

$$22mm < l_M = x + x_0 < 126mm$$

$$m_1 = 21.4g$$

$$m_2 = 23.95g$$

$$m_{ee} = 276.133g$$

$$M = 47.866$$

So that the $B = 0.0053$

$$\begin{bmatrix} \Delta \dot{x}_1(t) \\ \Delta \dot{x}_2(t) \\ \Delta \dot{x}_3(t) \\ \Delta \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \\ \Delta x_4(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 188.3 & 0 \\ 0 & 0 \\ 0 & 188.3 \end{bmatrix} \begin{bmatrix} \Delta \tau_1(t) \\ \Delta \tau_2(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \\ \Delta x_4(t) \end{bmatrix}$$

By Decomposition

$$\begin{bmatrix} \Delta \dot{x}_1(t) \\ \Delta \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 188.3 \end{bmatrix} \Delta \tau_1(t) \quad y(t) = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix}$$

$$\begin{bmatrix} \Delta \dot{x}_3(t) \\ \Delta \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_3(t) \\ \Delta x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 188.3 \end{bmatrix} \Delta \tau_2(t) \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix}$$

So we can only consider

$$\begin{bmatrix} \Delta \dot{x}_3(t) \\ \Delta \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_3(t) \\ \Delta x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 188.3 \end{bmatrix} \Delta \tau_2(t) \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix}$$

Discrete-time state space model

With sampling time $h = [0.004 \quad 0.0004 \quad 0.00004]$

$h = 0.004$

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} 1 & 0.004 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.001507 \\ 0.7533 \end{bmatrix} [\tau_2[k]]$$

$$y[k] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

$h = 0.0004$

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} 1 & 0.0004 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.00001507 \\ 0.07533 \end{bmatrix} [\tau_2[k]]$$

$$y[k] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

$h = 0.00004$

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} 1 & 0.00004 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.0000001507 \\ 0.007533 \end{bmatrix} [\tau_2[k]]$$

$$y[k] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

Analysis

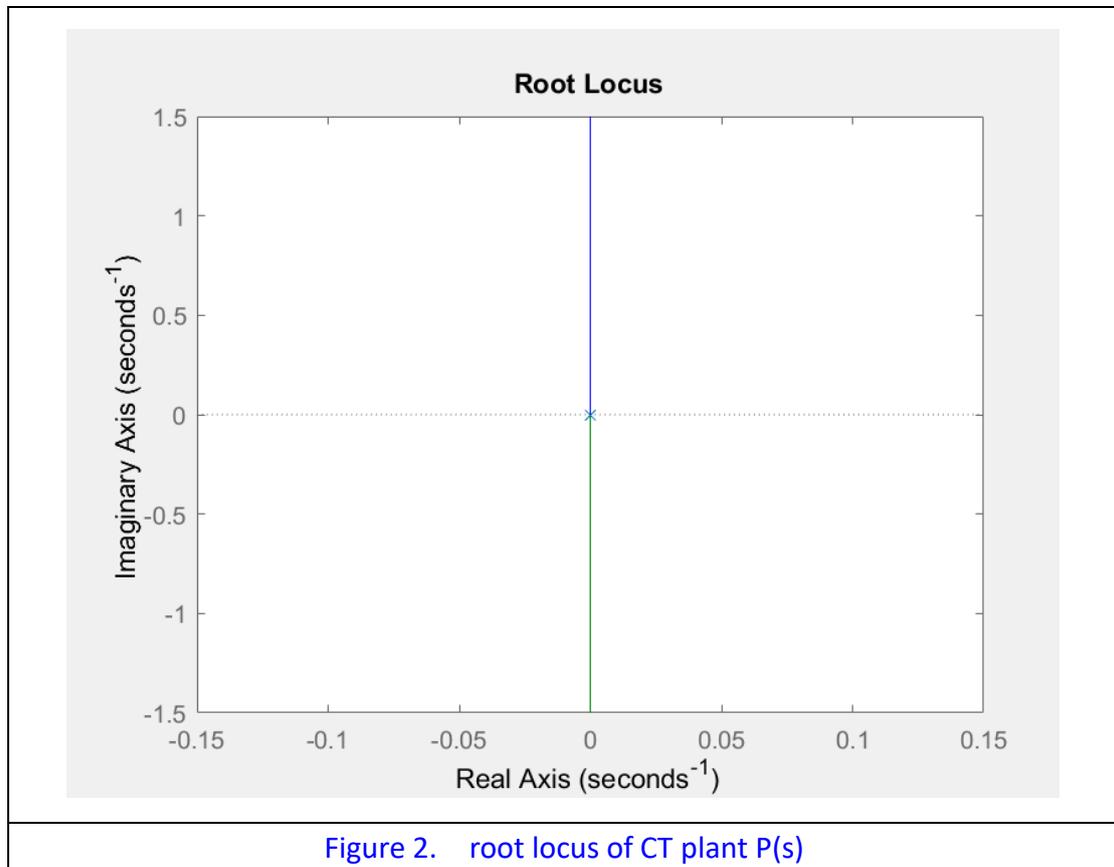
Continuous time

Stability:

We use root locus method to verified the CT system stability, the CT plant is

$P(s) = \frac{188.3}{s^2}$	(1)
----------------------------	-----

The root locus is



the system is marginal stable, and the step response is

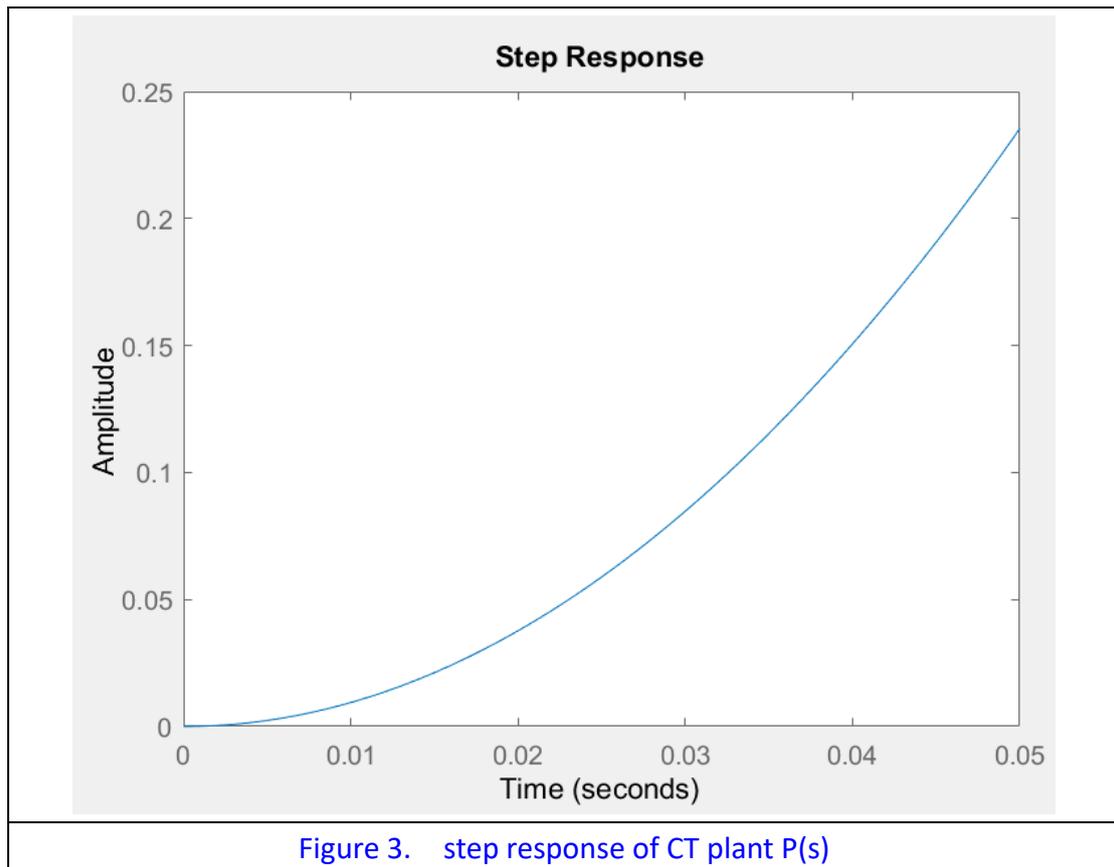


Figure 3. step response of CT plant P(s)

Controllability:

We verify the controllability matrix whether the rank of W_c is equal to n where $n = 2$

$W_c = [B \quad AB]$ $= \begin{bmatrix} 0 & 188.3 \\ 188.3 & 0 \end{bmatrix}$	(2)
---	-----

The rank of W_c is 2 so that the P(s) is controllable

Observability:

We verify the observability matrix whether the rank of W_o is equal to n where $n = 2$

$W_o = \begin{bmatrix} C \\ CA \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	(3)
--	-----

The rank of W_o is 2 so that the P(s) is observable

Discrete time

Stability:

We use root locus method to verified the DT system stability, the DT plant is

$h = 0.004$:

$P(z) = \frac{0.001507z + 0.001507}{z^2 - 2z + 1}$	(4)
--	-----

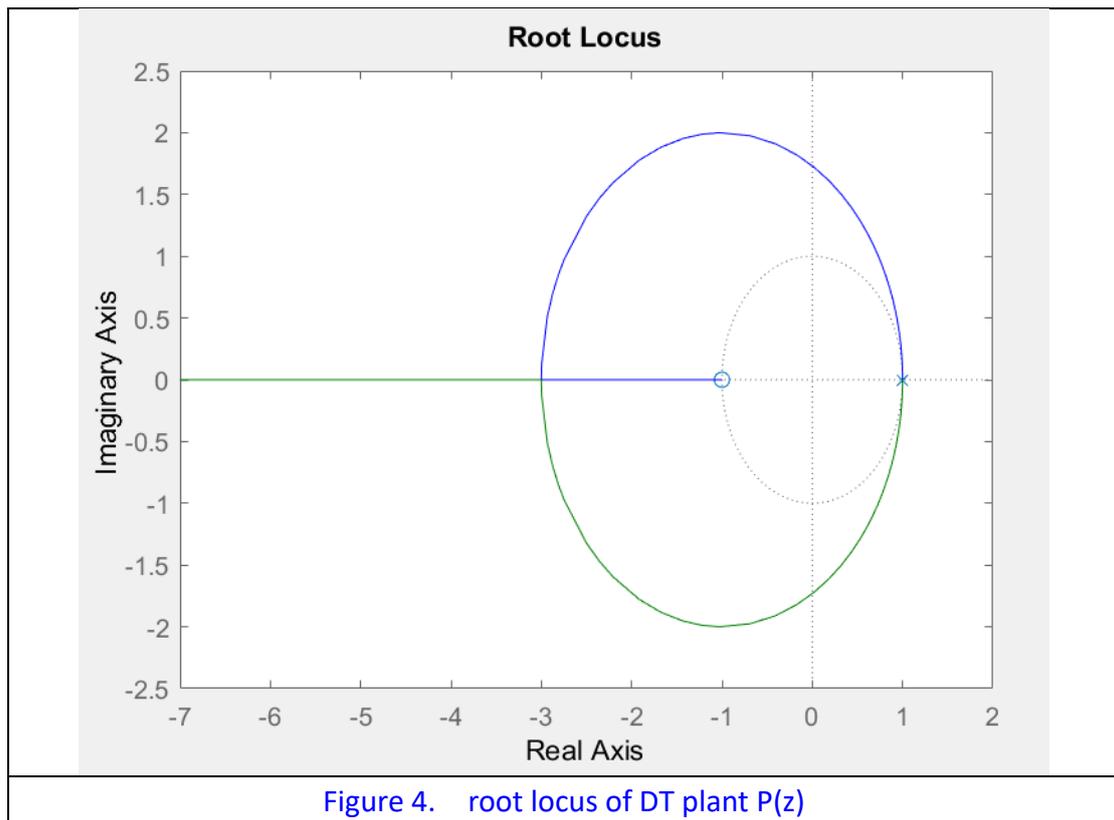
$h = 0.0004$:

$P(z) = \frac{1.507e - 05 z + 1.507e - 05}{z^2 - 2z + 1}$	(5)
---	-----

$h = 0.00004$:

$P(z) = \frac{1.507e - 07 z + 1.507e - 07}{z^2 - 2z + 1}$	(6)
---	-----

The root locus is



the system is marginal stable and the step response is

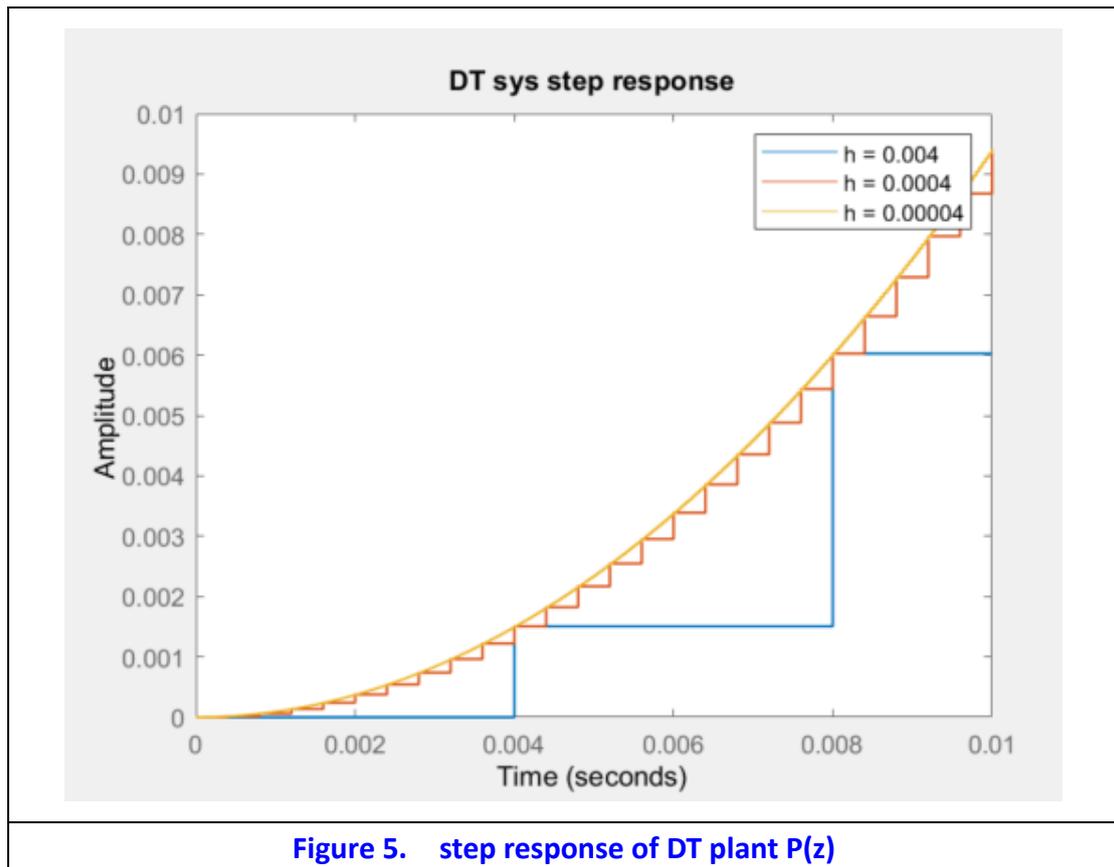


Figure 5. step response of DT plant P(z)

Controllability:

We verify the controllability matrix whether the rank of W_c is equal to n where $n = 2$

$W_c = [H \quad FH]$	(7)
----------------------	-----

We check the rank by matlab command **rank(W_c)** with different sampling time h

The rank of W_c are all equal to 2. As the result the P(z) is controllable.

Observability:

We verify the observability matrix whether the rank of W_o is equal to n where $n = 2$

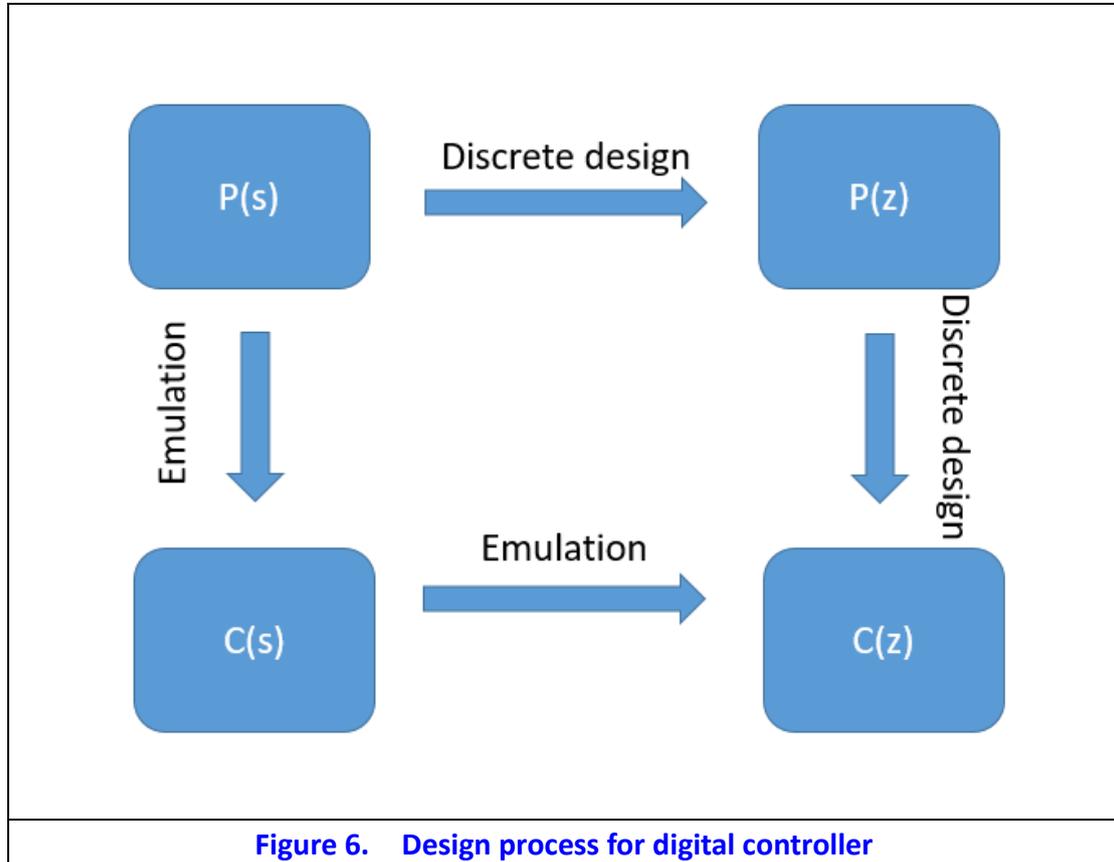
$W_o = \begin{bmatrix} C \\ CF \end{bmatrix}$	(8)
---	-----

We check the rank by matlab command **rank(W_o)** with different sampling time h

The rank of W_c are all equal to 2. As the result the P(z) is observable.

Design

In order to design a digital controller, we have two design methods – **Discrete design** and **Emulation**, the design process is as figure (5)



We will design our digital controller-pole placement method and PD controller by this two method and analysis the performance

Pole Placement

Discrete design

According spec, we can calculate the damping ratio and natural frequency using following equation

$$T_s \approx \frac{4.6}{\zeta \omega_n}$$
$$\zeta = \frac{-\ln(Mp)}{\sqrt{\pi^2 + \ln^2(Mp)}}$$

(9)

So our damping ratio and natural frequency and ideal model is

$\zeta = 0.8261 \quad \omega_n = 55.6843 \text{ rad/s}$ $G(s) = \frac{1}{s^2 + 92s + 3100.7}$	(10)
---	------

Base on

$\text{sampling time } h < \frac{1}{30 \frac{\omega_n}{2\pi}}$	(11)
--	------

Set $h = 0.0004 \text{ s}$

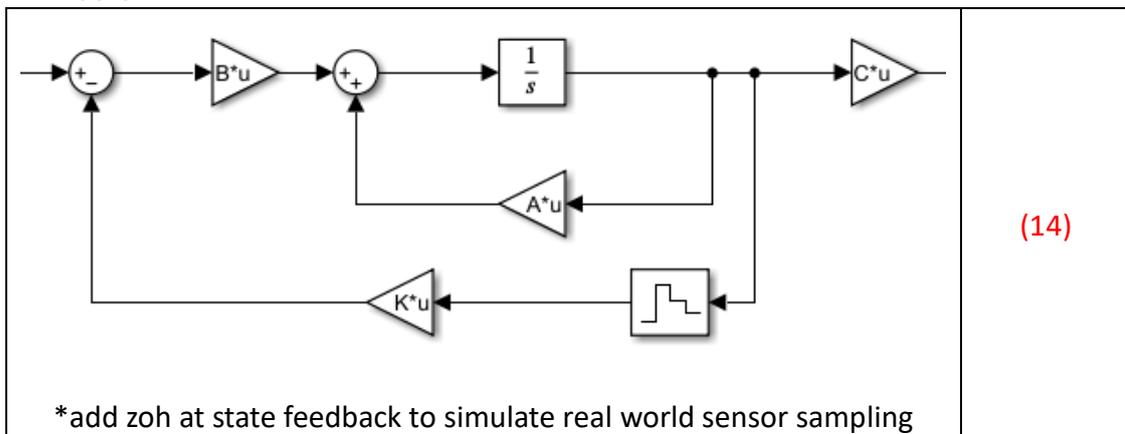
So can get $G(z)$ and ideal poles

$G(z) = \frac{1}{z^2 - 1.9634z + 0.9639}$ $\text{poles} = 0.9817 \pm 0.0123i$	(12)
---	------

We use pole placement method on DCS31-SSDesign-14 with our discrete model to get K

$\det(zI - (F - HK)) = z^2 - 1.9634z + 0.9639$ $[k_1 \ k_2] = [16.1659 \ 0.4829]$	(13)
---	------

Than apply $k_1 \ k_2$ on CT SS model



And the state feedback's step response is

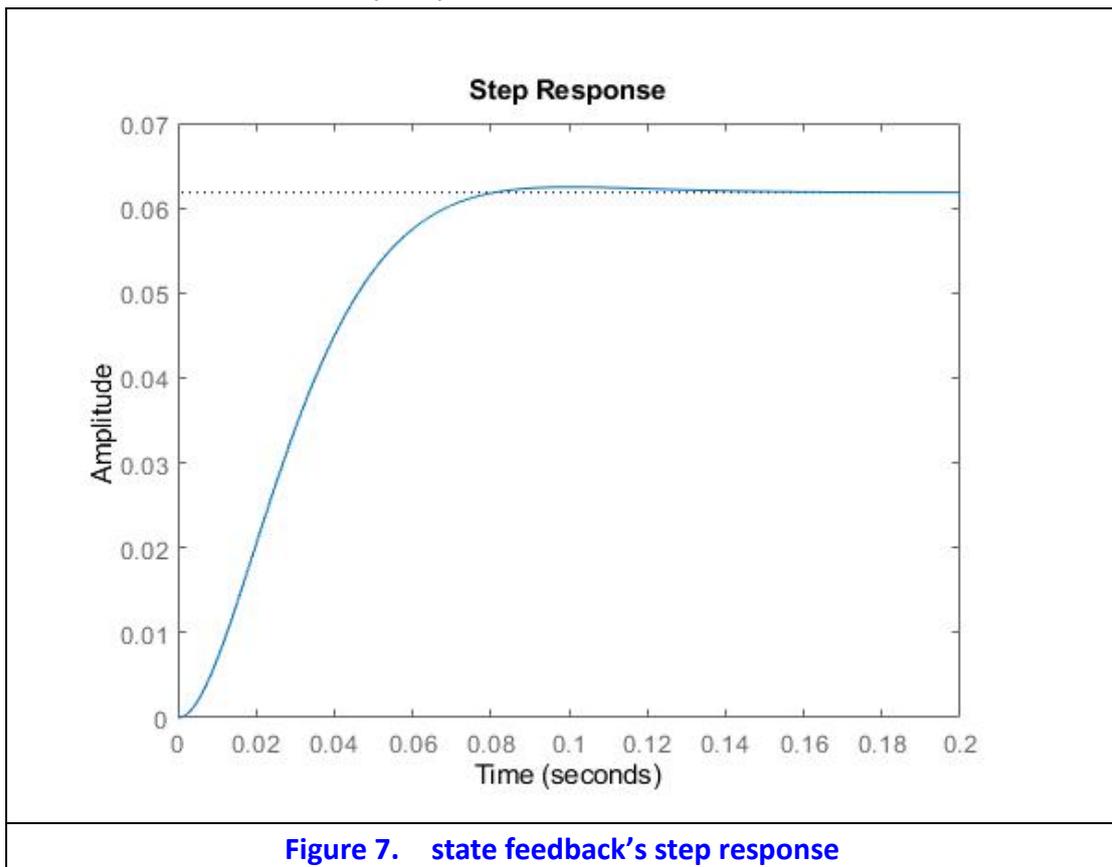
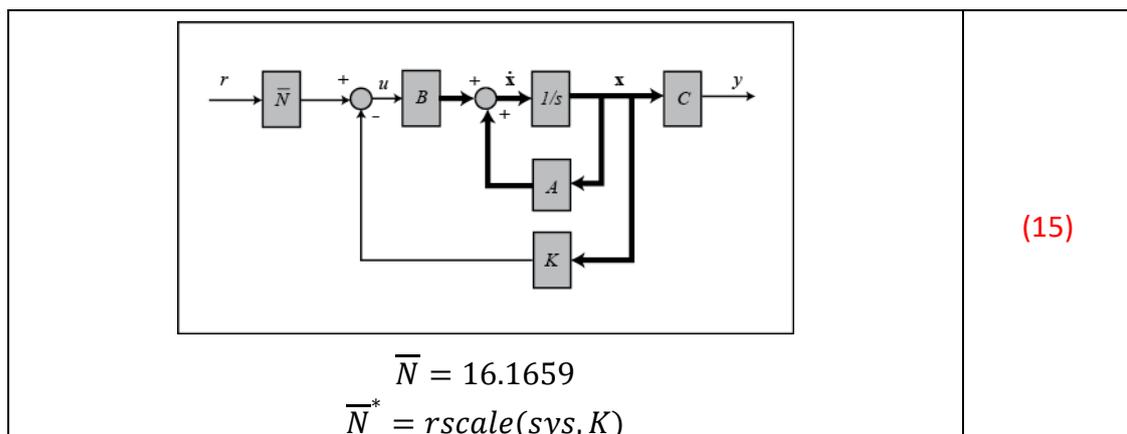


Figure 7. state feedback's step response

The steady state error is too large, we add a scaling factor after input as following block diagram and the \bar{N} can be calculate is due to the good knowledge of the system $G(s)$. The method was introduced on the website called Control Tutorials for Matlab and Simulink.



(15)

$$\bar{N} = 16.1659$$

$$\bar{N}^* = rscale(sys, K)$$

The step response with \bar{N} is

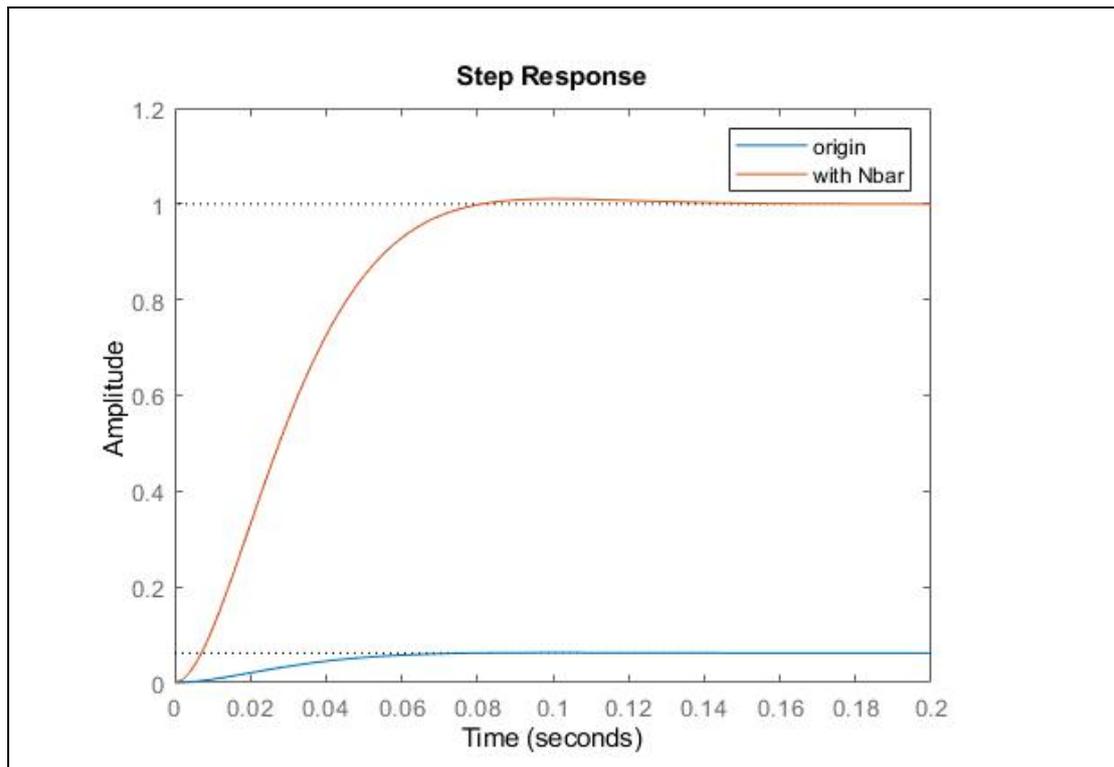


Figure 8. state feedback's step response with \bar{N}

Emulation

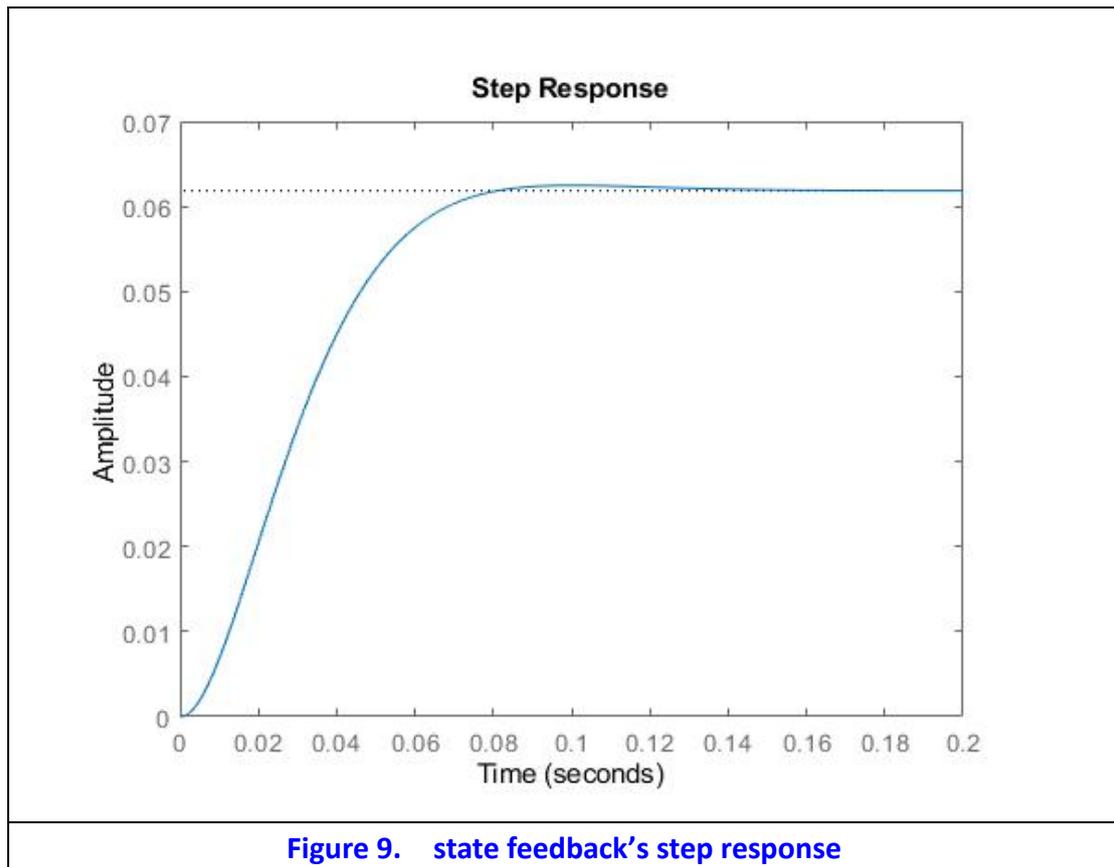
Mostly same as discrete design. But this time we place poles on CT

$\det(sI - (A - BK)) = s^2 + 92s + 3100.7$ $[k_1 \ k_2] = [16.4659 \ 0.4885]$	(16)
---	------

New scale factor come with CT sys and new [k1 k2]

$\bar{N} = 16.1659$	(17)
---------------------	------

And result is



PD controller

Emulation

The emulation design process is as following

$P(s) \rightarrow C(s) \rightarrow C(z)$	(18)
--	-------------

First, we design a CT controller to meet our specification

0 steady state error

settling time(t_s) in 0.1s

Overshoot (M_p) in 1%

According to the root locus for CT plant figure (1) we cannot achieve our goal by only P controller, so we apply PD controller

$C(s) = k_p + k_d s$	(19)
----------------------	-------------

The closed loop CT transfer function $H(s)$ is

$H(s) = \frac{PC}{1 + PC} = \frac{188.3(k_d s + k_p)}{s^2 + 188.3k_d s + 188.3k_p}$	(20)
---	------

To meet the t_s specification, we have the following approximately equation

$\sigma > \frac{4.6}{t_s}$ $\sigma > 46$	(21)
--	------

Where $-\sigma$ is real part location of $H(s)$ poles

To meet the M_p specification, we have the following approximately equation when

no zeros

$\zeta > \frac{\sqrt{(\ln M_p)^2}}{\sqrt{\pi^2 + (\ln M_p)^2}}$ $\zeta > 0.82$	(22)
--	------

Where ζ is damping ratio $H(s)$.

Now, we can get k_p and k_d

$k_p = 46.46$ $k_d = 0.4886$	(23)
------------------------------	------

And our closed loop CT system becomes

$H(s) = \frac{PC}{1 + PC} = 188.3 \frac{(0.4886s + 16.46)}{s^2 + 92s + 3101}$	(24)
---	------

The step response of $H(s)$ is

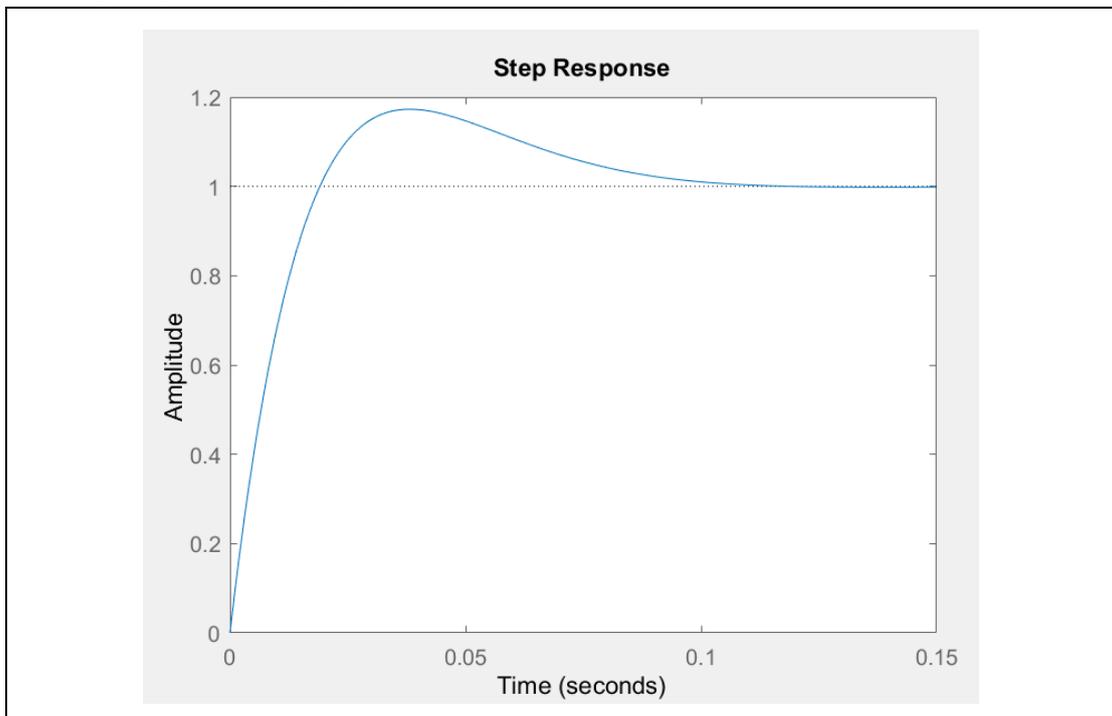


Figure 10. step response of $H(s)$

With

$$t_s = 0.0916s$$

$$M_p = 17\%$$

The t_s meet our specification, however, the M_p doesn't. That is because there is a zero in $H(s)$. The pole zero map is as following

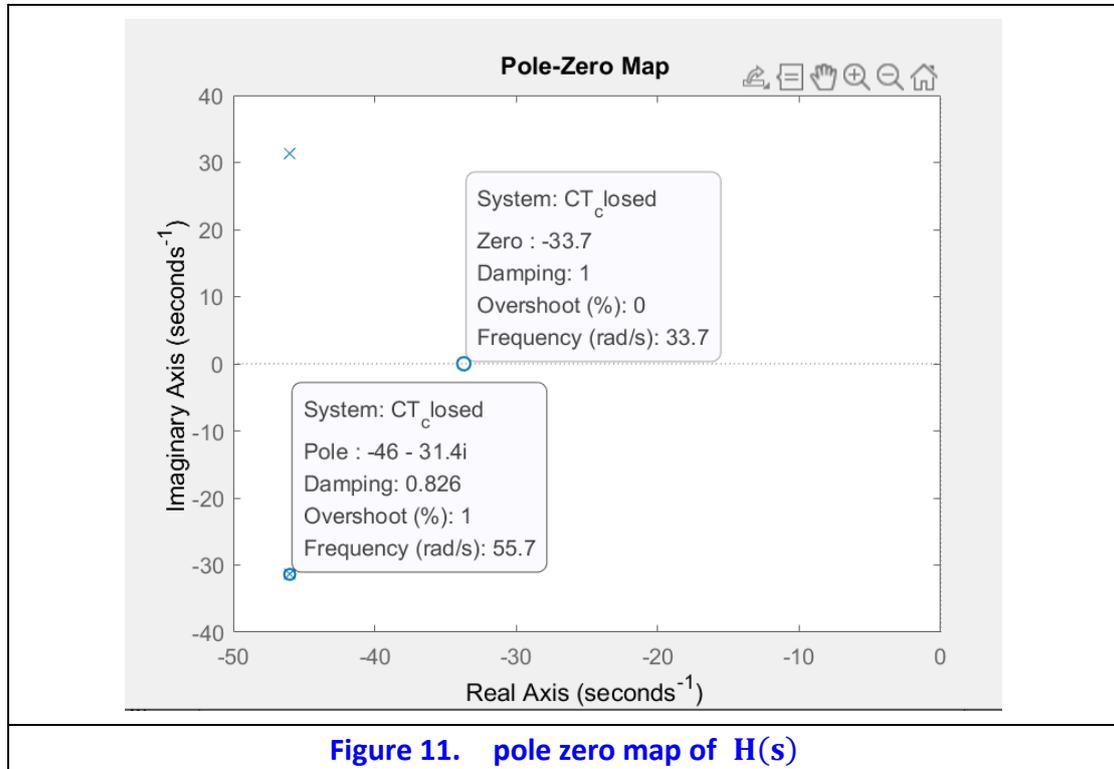


Figure 11. pole zero map of H(s)

To reduce the effectiveness of zero to this system, we tune the k_d in the purpose of shifting the zero to be close to original.

we gradually increase the k_d by multiples and observe the pole and zero location and step response of $H(s)$

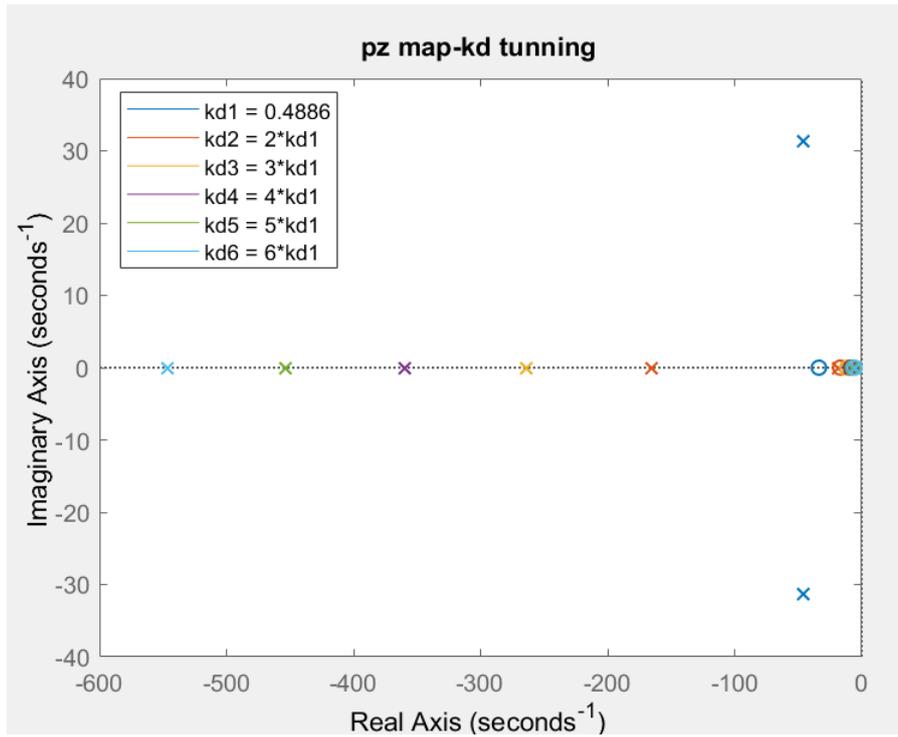


Figure 12. pole zero map for kd tuning

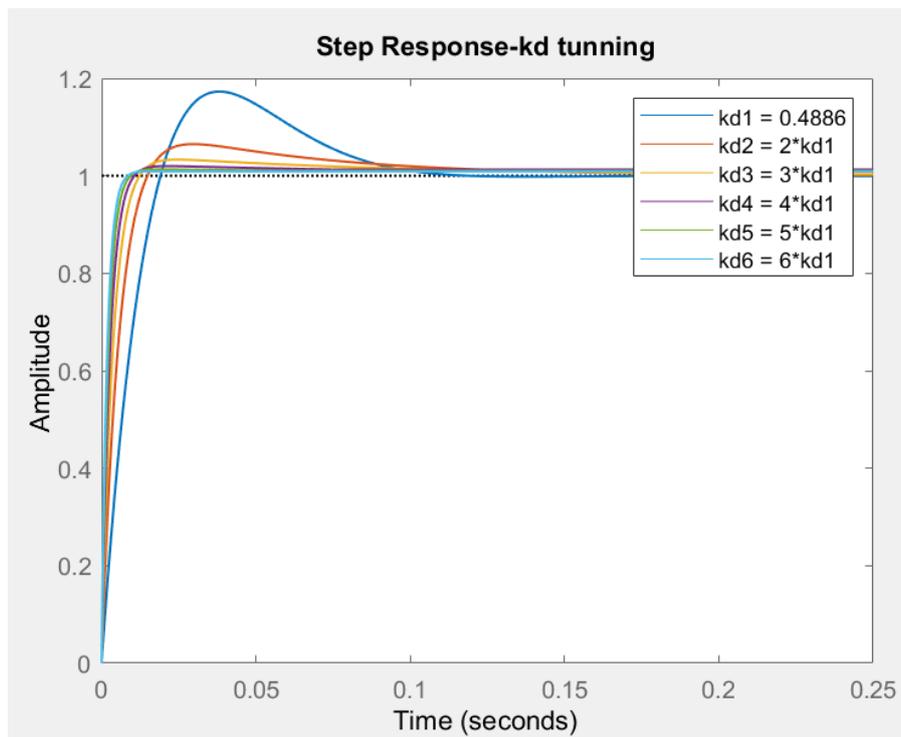


Figure 13. step response for kd tuning

When $k_d = 6 * k_a$, the pole zero map and the step response is

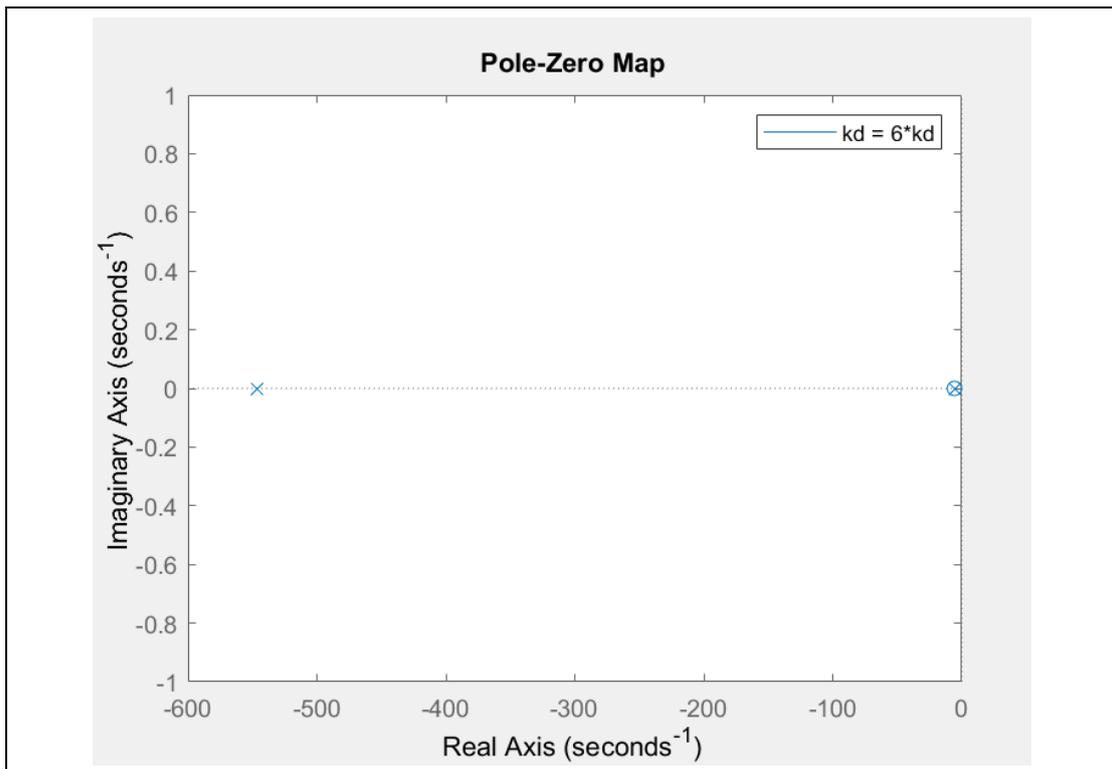


Figure 14. pole zero map of when $kd = 6kd$

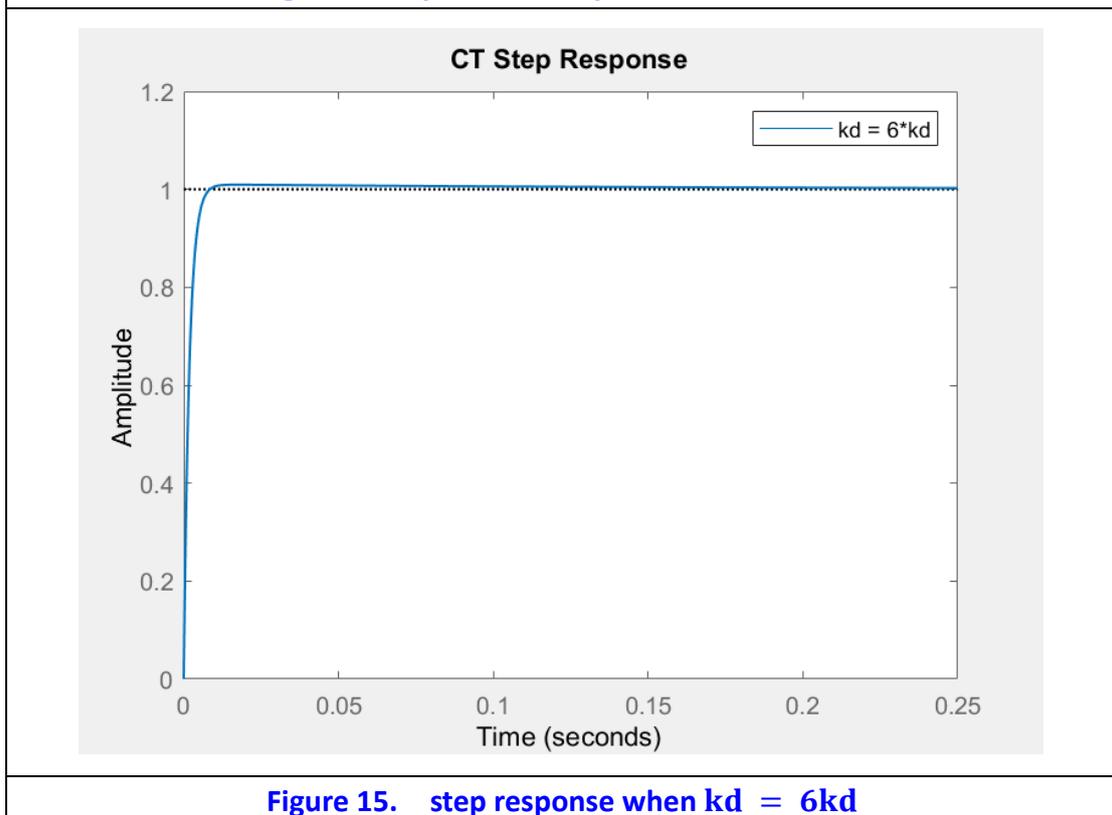


Figure 15. step response when $kd = 6kd$

With

$$t_s = 0.0064s$$

$$M_p = 0.9\%$$

The $H(s)$ is

$H(s) = 188.3 \frac{(2.93s + 16.46)}{s^2 + 552s + 3101}$	(25)
--	------

Next, we convert the CT controller to DT controller by Tustin method with 30 times natural frequency(w_n) as sampling frequency and sampling time $h = 0.0004$

We also compare the performance when $h = 0.004$ and $h = 0.00004$

$h = 0.004$:

$C(z) = \frac{1482z - 1449}{z + 1}$	(26)
-------------------------------------	------

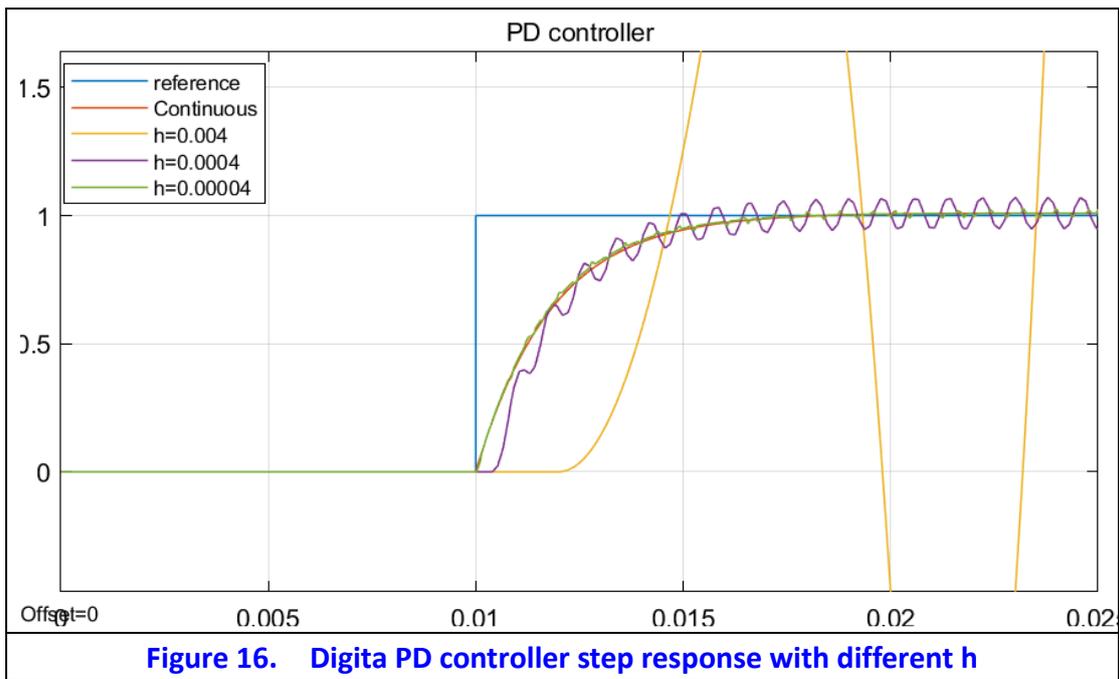
$h = 0.0004$:

$C(z) = \frac{1.467e04 z - 1.464e04}{z + 1}$	(27)
--	------

$h = 0.00004$:

$C(z) = \frac{1.466e05 z - 1.466e05}{z + 1}$	(28)
--	------

And the simulation result is



Discussion and conclusion

Controller – SS pole placement v.s. PD

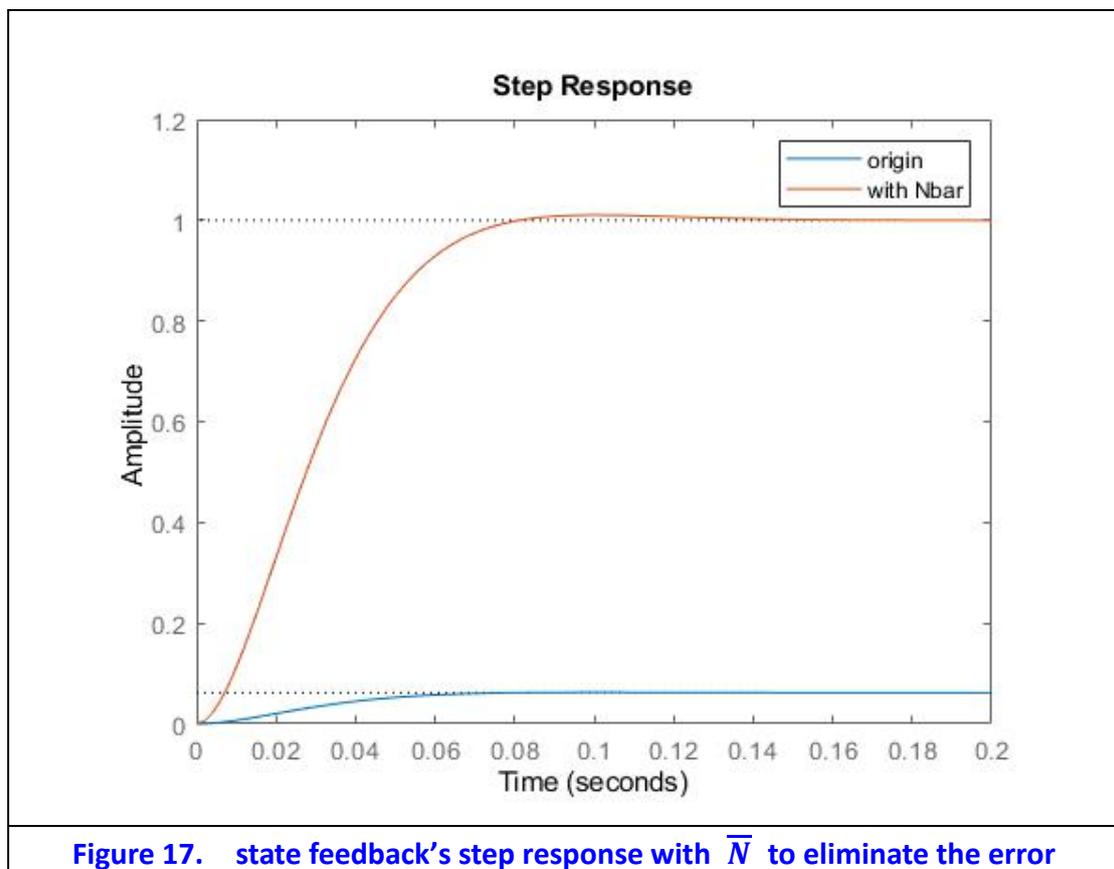
In the designing process we find out some characteristic of the two controller

SS pole placement

- Need have plant model first
- place pole in one step
- Need deal with steady error (figure 15)

PD

- Don't need well knowledge about plant
- Place pole will cause zero to generate the overshoot (figure 16)
- No steady error



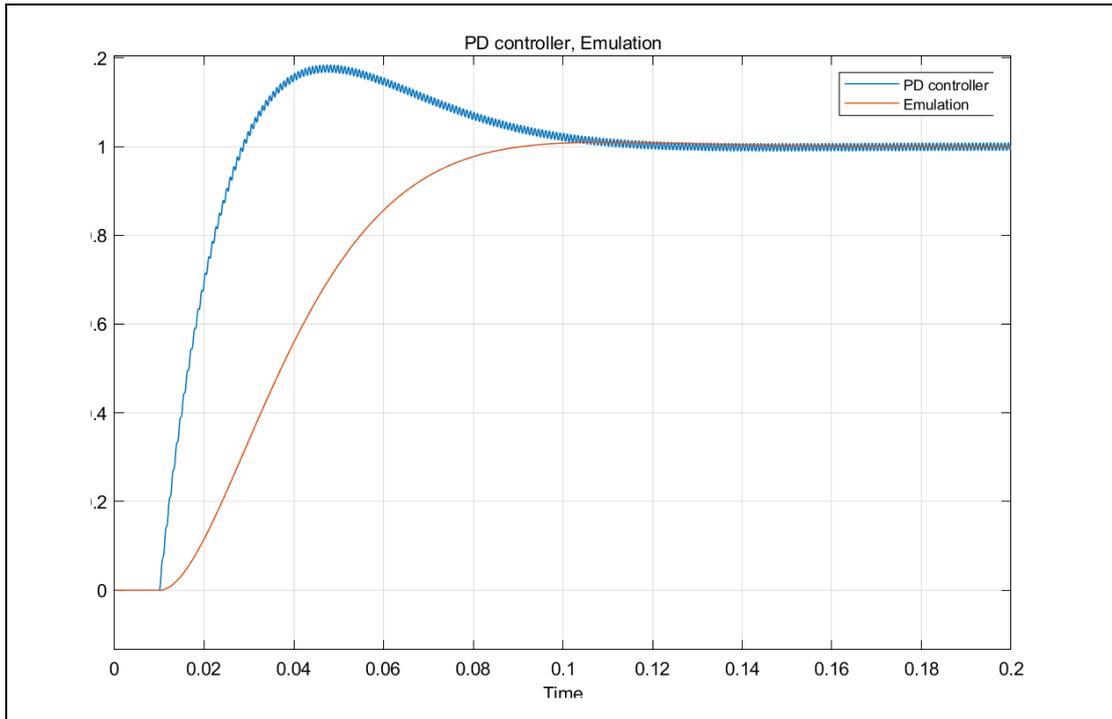


Figure 18. the same w_n and ζ of different controller step response

Design process – Emulation v.s. Discrete

We take state feedback poleplacement method as example

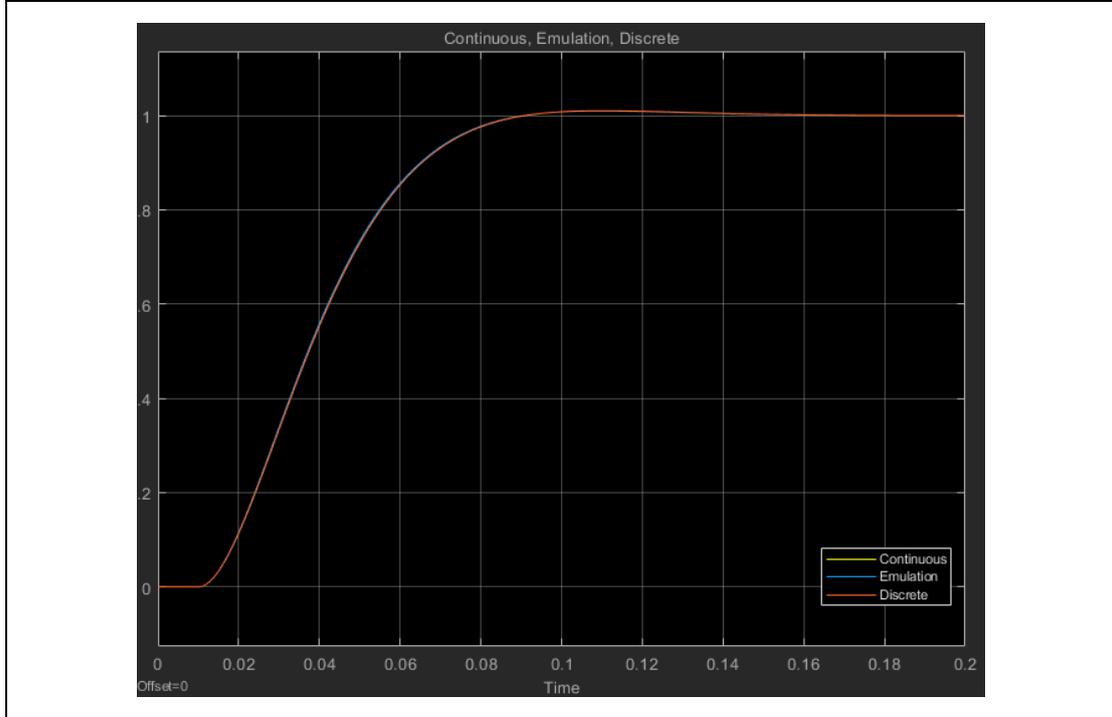


Figure 19. step response when $k_d = 6k_d$

We plot the step response of continuous time, emulation and digital controller on the same diagram. Zoom in the overshoot area

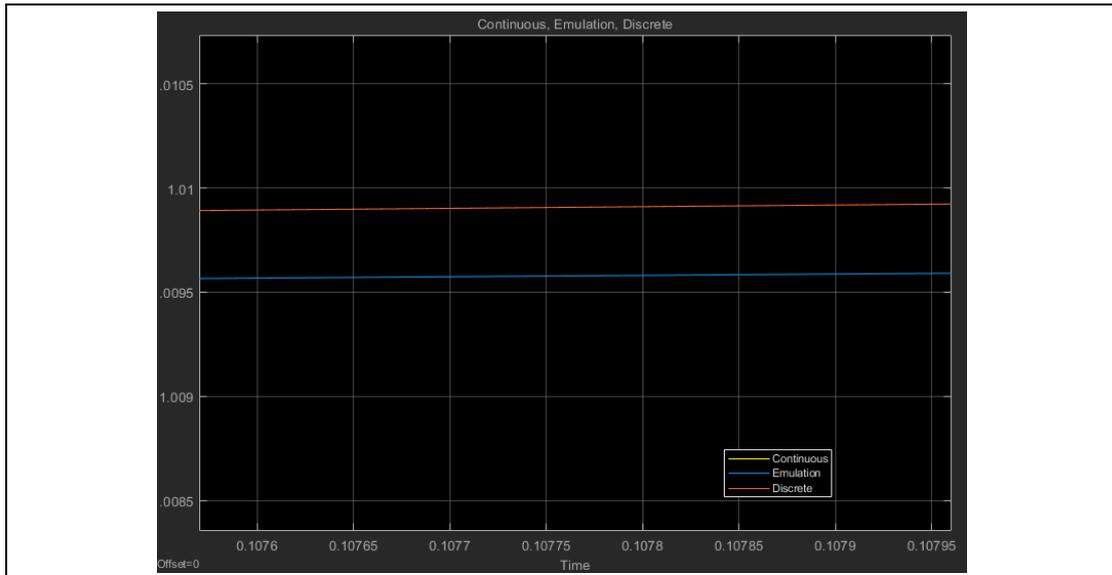


Figure 20. step response when $k_d = 6k_d$

We can find out the difference. The discrete method controller almost overlap with the continuous time controller. Emulation method, by contrast, has larger difference with continuous time controller. As the result, the digital design has better performance in this part.

Sampling time- 0.004 v.s. 0.0004 v.s. 0.00004

We take PD controller with emulation method as example

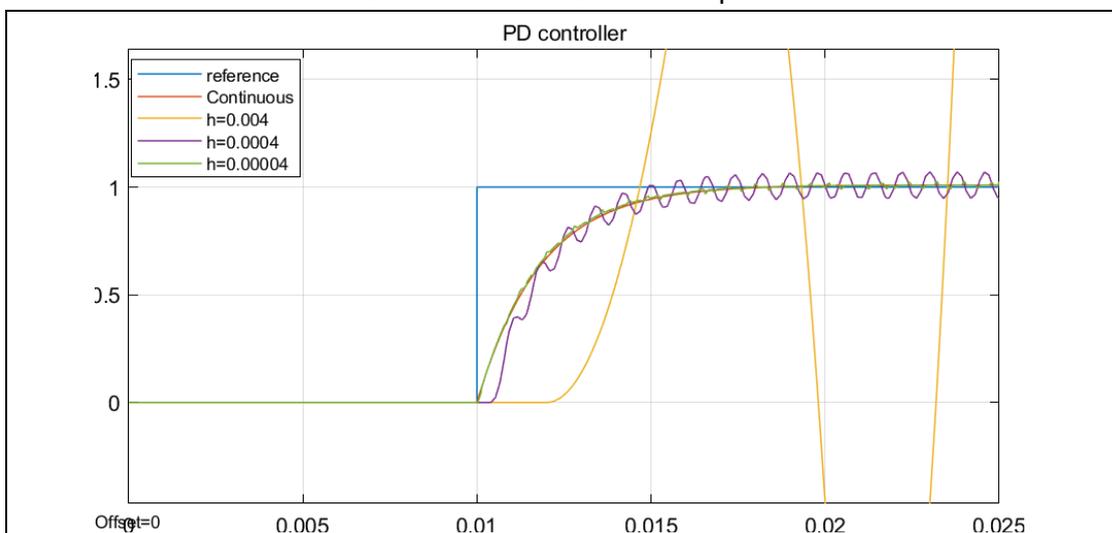


Figure 21. Digital PD controller step response with different h

According to the experience ($30 \text{ times } w_n$), we choose $h = 0.0004$ (purple) as sampling time, the result is not bad; If we choose $h = 0.00004$ (green) as sampling time, the result nearly overlap with continuous time controller (orange). On the other hand, if we choose $h = 0.004$ (yellow), the system will diverge due to the low sampling time.

References

- [1: Astrom, Wittenmark 1997]
Computer-Controlled Systems: Theory and Design, 3rd Ed. (1997)
- [2: Gene F. Franklin, J. David Powell & Abbas Emami-Naeini (1997)]
Feedback Control of Dynamic Systems Sixth Edition
- [3: S. Manna, S. Shaw and A. K. Akella (2020)]
S. Manna, S. Shaw and A. K. Akella, "Design of Higher-Order Regulator System using Pole-Placement Technique," *2020 International Conference on Computational Intelligence for Smart Power System and Sustainable Energy (CISPSSE)*, 2020, pp. 1-4, doi: 10.1109/CISPSSE49931.2020.9212270.
- [4: Miguel Ayala Botto, Robert Babuška, José Sá da Costa (2002)]
Miguel Ayala Botto, Robert Babuška, José Sá da Costa, DISCRETE-TIME ROBUST POLE-PLACEMENT DESIGN THROUGH GLOBAL OPTIMIZATION, *IFAC Proceedings Volumes*, Volume 35, Issue 1, 2002, Pages 343-348,
- [5: Quan, Quan, Lu Jiang, and Kai-Yuan Cai (2014)]
Discrete-Time Output-Feedback Robust Repetitive Control for a Class of Nonlinear Systems by Additive State Decomposition
- [6: H. D. Raut, A. Singh and M. D. Patil (2009)]
H. D. Raut, A. Singh and M. D. Patil, "Design of digital controller using pole placement method," *2009 International Conference on Control, Automation, Communication and Energy Conservation*, 2009, pp. 1-5.
- [7: M. Kocur, S. Kozak and B. Dvorscak (2014)]
M. Kocur, S. Kozak and B. Dvorscak, "Design and Implementation of FPGA - digital based PID controller," *Proceedings of the 2014 15th International Carpathian Control Conference (ICCC)*, 2014, pp. 233-236, doi: 10.1109/CarpathianCC.2014.6843603.
- [8: H. Omran, L. Hetel, J. Richard and F. Lamnabhi-Lagarrigue (2012)]
Stability of bilinear sampled-data systems with an emulation of static state feedback
- [9: Dawn Tilbury, Bill Messner, Rick Hill, JD Taylor (2021)]
Control Tutorials for MATLAB and Simulink
- [10: CHEN Xiang, HUANG Long, YANG Yang (2017)]
CHEN Xiang, HUANG Long, YANG Yang. Gravity compensation of an intraocular surgery robot based on computed torque method[J]. *JOURNAL OF BEIJING UNIVERSITY OF AERONAUTICS AND A*, 2017, 43(6): 1231-1238.